A.1 Algebra Essentials

1 Work with Sets

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D = \{x \mid x \text{ is a digit}\}$

$D = \{\quad x \quad | \quad x \text{ is a digit}\}$

Read as "D is the set of all x such that x is a digit."

EXAMPLE 1 Using Set-builder Notation and the Roster Method

(a) $E = \{x \mid x \text{ is an even digit}\} = \{2, 4, 6, 8\}$

(b) $O = \{x \mid x \text{ is an odd digit}\} = \{1, 3, 5, 7, 9\}$

If every element of a set A is also an element of a set B, then A is a **subset** of B, which is denoted $A \subseteq B$. If two sets A and B have the same elements, then A **equals** B, which is denoted $A = B$.

For example, $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$ and $\{1, 2, 3\} = \{2, 3, 1\}$.

A B

DEFINITION

If A and B are sets, the **intersection** of A with B, denoted $A \cap B$, is the set consisting of elements that belong to both A and B. The **union** of A with B, denoted $A \cup B$, is the set consisting of elements that belong to either A or B, or both.

EXAMPLE 2 **Finding the Intersection and Union of Sets**

Let $A = \{1, 3, 5, 8\}$, $B = \{3, 5, 7\}$, and $C = \{2, 4, 6, 8\}$. Find:

(a) $A \cap B$ (b) $A \cup B$ (c) $B \cap (A \cup C)$

a) $A \cap B = \{3, 5\}$

b) $A \cup B = \{1, 3, 5, 7, 8\}$

c) $A \cup C = \{1, 2, 3, 4, 5, 6, 8\}$; $B \cap (A \cup C) = \{3, 5\}$

DEFINITION

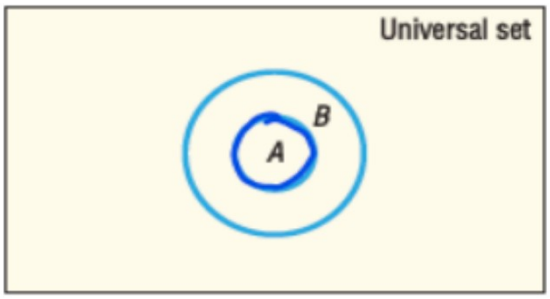
If A is a set, the **complement** of A, denoted $\bar{A}$, is the set consisting of all the elements in the universal set that are not in A.*

EXAMPLE 3 **Finding the Complement of a Set**

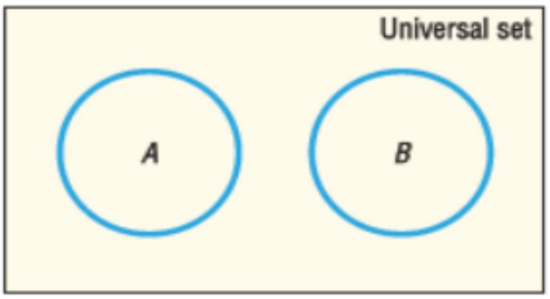
If the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and if $A = \{1, 3, 5, 7, 9\}$, then

find $\bar{A}$

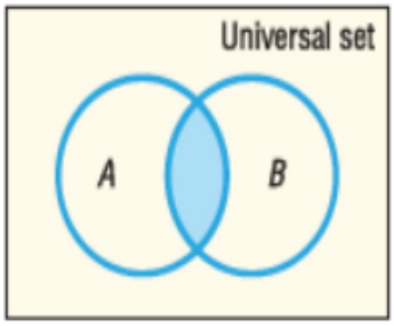
$\bar{A} = \{2, 4, 6, 8\}$



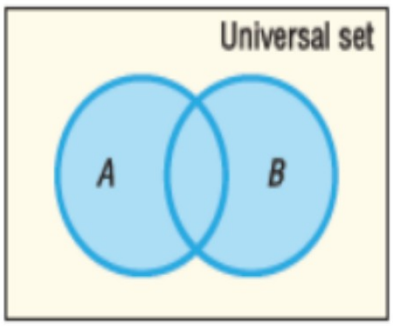
(a) $A \subseteq B$
subset



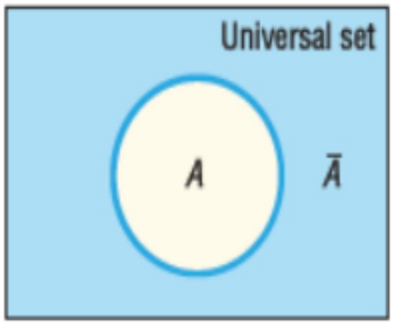
(b) $A \cap B = \varnothing$
disjoint sets



(a) $A \cap B$
intersection



(b) $A \cup B$
union



(c) $\bar{A}$
complement

U = *universal set* = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 4, 5, 9\}$,

$B = \{2, 4, 6, 7, 8\}$, *and* $C = \{1, 3, 4, 6\}$

find

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cap B = \{4\}$

$(A \cup B) \cap C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 3, 4, 6\}$
$= \{1, 3, 4, 6\}$

$\bar{A} = \{0, 2, 6, 7, 8\}$

$\overline{A} \cup \overline{B}$

$\overline{A} = \{0, 2, 6, 7, 8\}$

$\overline{B} = \{0, 1, 3, 5, 9\}$

$\overline{A} \cup \overline{B} = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$

$\overline{B \cup C}$

$B \cup C = \{1, 2, 3, 4, 6, 7, 8\}$

$\overline{B \cup C} = \{0, 5, 9\}$

Real Numbers

Real numbers are represented by symbols such as

$$25,\ 0,\ -3,\ \frac{1}{2},\ -\frac{5}{4},\ 0.125,\ \sqrt{2},\ \pi,\ \sqrt[3]{-2},\ 0.666\ldots$$

The set of **counting numbers**, or **natural numbers**, contains the numbers in the set $\{1, 2, 3, 4, \ldots\}$. (The three dots, called an **ellipsis**, indicate that the pattern continues indefinitely.) The set of **integers** contains the numbers in the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. A **rational number** is a number that can be expressed as a *quotient* $\frac{a}{b}$ of two integers, where the integer b cannot be 0. Examples of rational numbers are $\frac{3}{4}, \frac{5}{2}, \frac{0}{4}$, and $-\frac{2}{3}$. Since $\frac{a}{1} = a$ for any integer a, every integer is also a rational number. Real numbers that are not rational are called **irrational**. Examples of irrational numbers are $\sqrt{2}$ and π (the Greek letter pi), which equals the constant ratio of the circumference to the diameter of a circle.

N = Natural = $\{1, 2, 3, 4 \ldots\}$

الطبيعية

whole = $\{0, 1, 2, 3, 4 \ldots\} = W$

الكلية

integer = $Z = \{0, \mp 1, \mp 2, \mp 3 \ldots\}$

الصحيحة

$N \subseteq W \subseteq Z$

Rational = $\{\frac{a}{b} ; b \neq 0\}$

$\frac{1}{3} = 0,333\ldots$ $\quad \frac{7}{2} = 3.5$ $\quad \frac{8}{2} = 4$ $\quad \frac{3}{5}$

فاصلة منتهية

*) irrational

غير نسبي

$\pi = 3,14\ldots$ $\quad \sqrt{2}$ $\quad 3.214\ldots$

$e = 2,71\ldots$ $\quad \sqrt{5}$ $\quad 7.43\ldots$

$\sqrt{7}$

ex The number $\sqrt{2}$ is

1) Natural
2) integer
3) rational
4) irrational

ex The number $\frac{3}{5}$ is

1) Natural
2) integer
3) rational
4) irrational

Distributive Property

$$a \cdot (b + c) = ab + ac$$

ex : $3(x+6) = 3x + 18$

ex $2(x-5) = 2x - 10$

Zero-Product Property

If $ab = 0$, then either $a = 0$ or $b = 0$ or both equal 0.

ex: $x \cdot y = 0 \Rightarrow x = 0$ or $y = 0$

In Problems 35–40, write each statement as an inequality.

35. x is positive

36. z is negative

38. y is greater than -5

39. x is less than or equal to 1

35) x is positive $= \{x / x > 0\}$

36) z is negative $= \{z / x < 0\}$

38) y is greater than $-5 = \{y / y > -5\}$

39) x is less than or equal to 1

$= \{x / x \leq 1\}$

In Problems 45–50, use the given real number line to compute each distance.



45. $d(C, D)$ **46.** $d(C, A)$ **47.** $d(D, E)$ **48.** $d(C, E)$

$d(C,D) = |C-D| = |D-C|$

$d(C,D) = |0-1| = |-1| = +1$

$d(C,A) = |C-A| = |0-(-3)| = |3| = 3$

$d(D,E) = |D-E| = |1-3| = |-2| = 2$

$d(C,E) = |C-E| = |0-3| = |-3| = 3$

evaluate each expression if $x = -2$ and $y = 3$.

$x + 2y$ DS1 $= (-2) + 2(3) = -2 + 6 = +4$

$5xy + 2 = 5(-2)(3) + 2 = -30 + 2 = -28$

$\frac{3x + 2y}{2 + y} = \frac{3(-2) + 2(3)}{2+3} = \frac{-6+6}{5} = \frac{0}{5} = 0$

example : evaluate if $x=-5$
$y=-8$

1) $|2x+3y|$ 2) x^2+y^2

Sol

1) $|2x+3y| = |2(-5)+3(-8)| = |-10-24|$
$= |-34| = 34$

2) $(-5)^2+(-8)^2 = 25+64 = 89$