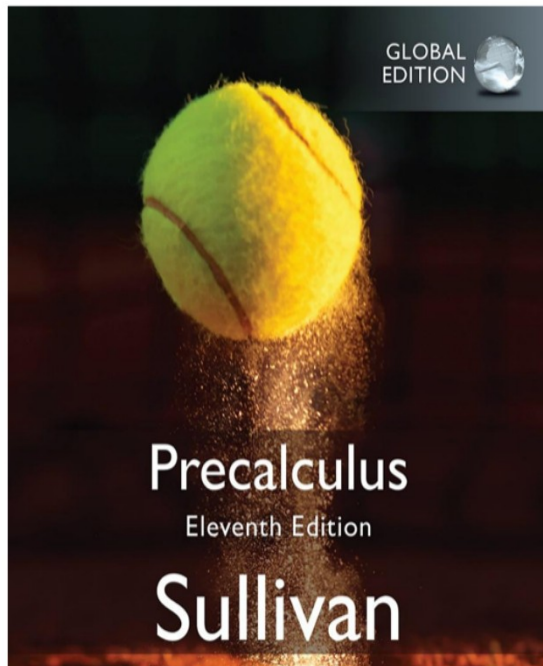


Precalculus

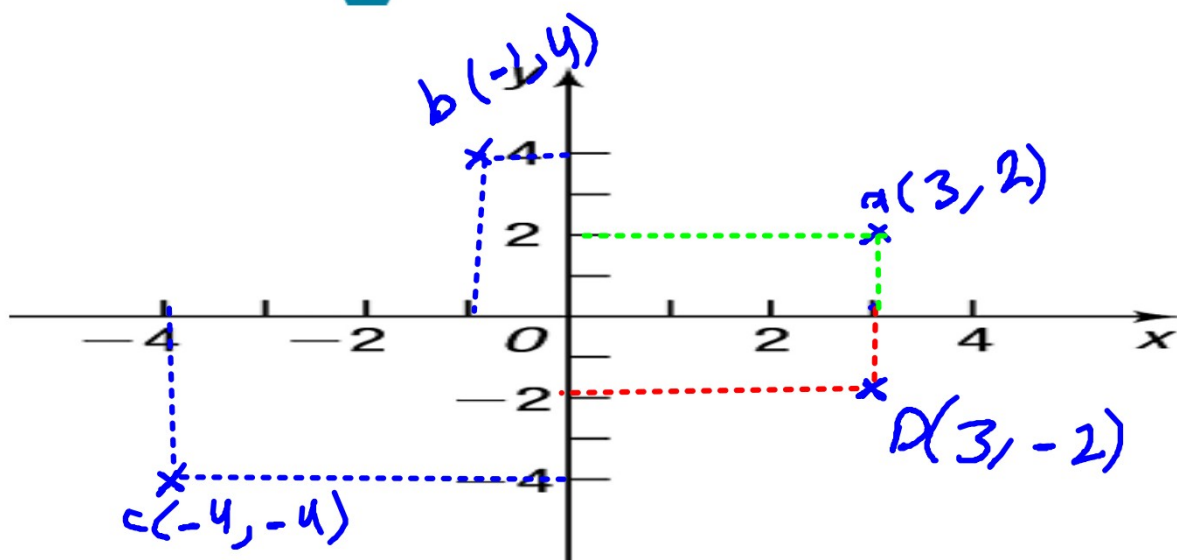
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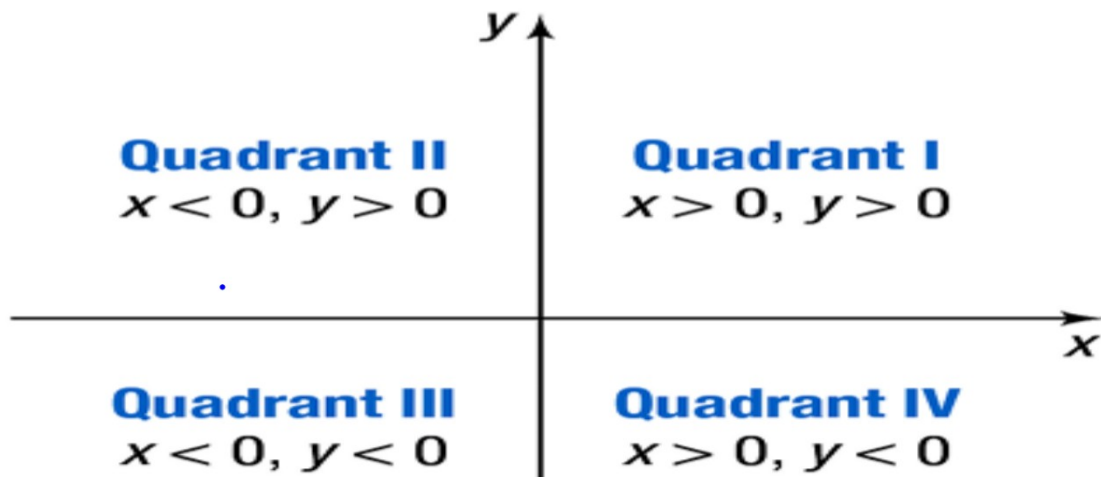


Chapter 1 Graphs

Section 1.1 The Distance and Midpoint Formulas

Rectangular Coordinates





The point $(-3, 5)$ is in quadrant

The point if $x > 0, y < 0$ is in quadrant

- A) Q I
- B) Q II**
- C) Q III
- D) Q IV

- A) Q I
- B) Q II
- C) Q III
- D) Q IV**

Theorem 1

(2) PS

Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance d between the points $(-4, 5)$ and $(1, 3)$.

x_1, y_1

x_2, y_2

$$d(a, b) = \sqrt{(-4-1)^2 + (5-3)^2} = \sqrt{(-5)^2 + (2)^2} \\ = \sqrt{25 + 4} = \sqrt{29}$$

ex: Find The distance between The points $a(5, 4)$, $b(-3, -2)$

$$d(a, b) = \sqrt{(-3-5)^2 + (-2-4)^2} = \sqrt{(-8)^2 + (-6)^2} \\ = \sqrt{64 + 36} = \sqrt{100} = 10$$

ex: Find The distance between $a(-1, -3)$, $b(-4, -6)$

x_1, y_1 x_2, y_2

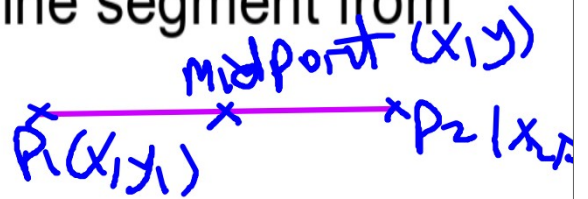
$$d(a, b) = \sqrt{(-4-(-1))^2 + (-6-(-3))^2} \\ = \sqrt{(-4+1)^2 + (-6+3)^2} = \sqrt{(-3)^2 + (-3)^2} \\ = \sqrt{9 + 9} = \sqrt{18} \\ = \sqrt{9 \times 2} = \sqrt{9} \sqrt{2} \\ = 3\sqrt{2}$$

Theorem 2

Midpoint Formula

The midpoint $M = (x, y)$ of a line segment from

$P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is



$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example

Find the midpoint of the line segment from

$$P_1(3, 7) \quad P_2(5, 1)$$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$\text{midpoint } (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{3+5}{2}, \frac{7+1}{2} \right) = \left(\frac{8}{2}, \frac{8}{2} \right) = (4, 4)$$

Example

Find the midpoint of the line segment from

$$P_1 = (\underset{x_1}{-4}, \underset{y_1}{4}) \text{ to } P_2 = (\underset{x_2}{4}, \underset{y_2}{2}).$$

$$\begin{aligned} \text{midpoint} &= \left(\frac{-4+4}{2}, \frac{4+2}{2} \right) \\ &= \left(\frac{0}{2}, \frac{6}{2} \right) = (0, 3) \end{aligned}$$

Example

Find the midpoint of the line segment from

$$P_1(\underset{x_1}{-5}, \underset{y_1}{-6}) \quad P_2(\underset{x_2}{-7}, \underset{y_2}{-12})$$

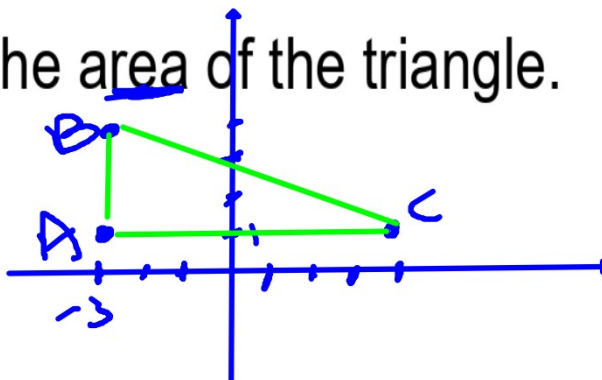
$$\begin{aligned} \text{midpoint} &= \left(\frac{-5+(-7)}{2}, \frac{-6+(-12)}{2} \right) \\ &= \left(\frac{-12}{2}, \frac{-18}{2} \right) = (-6, -9) \end{aligned}$$

Example

Find the midpoint of the line segment from

Consider three points $A = (-3, 1)$, $B = (-3, 4)$, and $C = (4, 1)$.

- (a) Plot each point and form the triangle ABC .
- (b) Find the length of each side of the triangle.
- (c) Verify that the triangle is a right triangle.
- (d) Find the area of the triangle.



$$2) d(A, B) = \sqrt{(-3 - (-3))^2 + (4 - 1)^2} = \sqrt{(-3 + 3)^2 + (3)^2} \\ = \sqrt{9} = 3$$

$$d(B, C) = \sqrt{(-3 - 4)^2 + (4 - 1)^2} = \sqrt{(-7)^2 + (3)^2} \\ = \sqrt{49 + 9} = \sqrt{58}$$

$$d(A, C) = \sqrt{(-3 - 4)^2 + (1 - 1)^2} = \sqrt{49 + (0)^2} = \sqrt{49} = 7$$

$$3) (BC)^2 = (AB)^2 + (AC)^2$$

$$(BC)^2 = (\sqrt{58})^2 = 58$$

$$(AB)^2 = (3)^2 = 9$$

$$(AC)^2 = (7)^2 = 49$$

$$\Rightarrow 9 + 49 = 58$$

right angle

$$4) \text{ area of triangle} = \frac{1}{2} \underline{b} \cdot \underline{h}$$

$$= \frac{1}{2} (3)(7) = \frac{21}{2} = 10.5$$