# Chapter 10.1
# Limits

## Definition

The limit of $f(x)$ as $x$ approaches $a$ is the number $L$, written

$$\lim_{x \to a} f(x) = L$$

provided that we can make the values $f(x)$ as close as we like to $L$, and keep that close, by taking $x$ sufficiently them close to, but different from, $a$.

number, we say that If there is no such the limit of $f(x)$ as $x$ approaches $a$ does not exist.

$\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$



## Examples

**3.** For the function $f$ graphed in the accompanying figure, find

(a) $\lim_{x \to 3^-} f(x) = -1$ (b) $\lim_{x \to 3^+} f(x) = 3$

(c) $\lim_{x \to 3} f(x) =$ DNE (d) $f(3) = -1$

D.N.E

E $f(2) = 3$

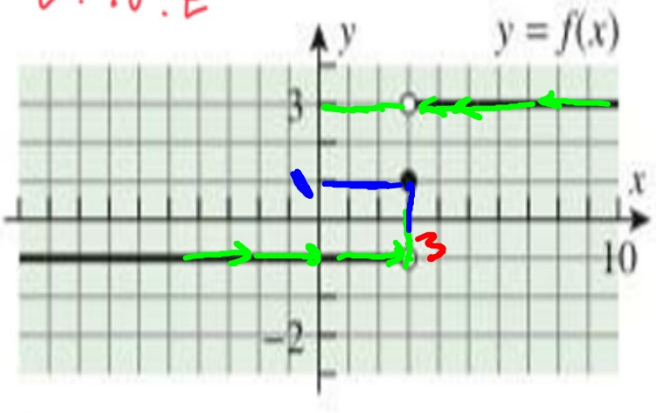


Figure Ex-3

**4.** For the function $f$ graphed in the accompanying figure, find

(a) $\lim_{x \to 2^-} f(x) = 2$ (b) $\lim_{x \to 2^+} f(x) = 0$

(c) $\lim_{x \to 2} f(x) =$ DNE (d) $f(2) = 2$

D.N.E

* دائم اختار المغلقة

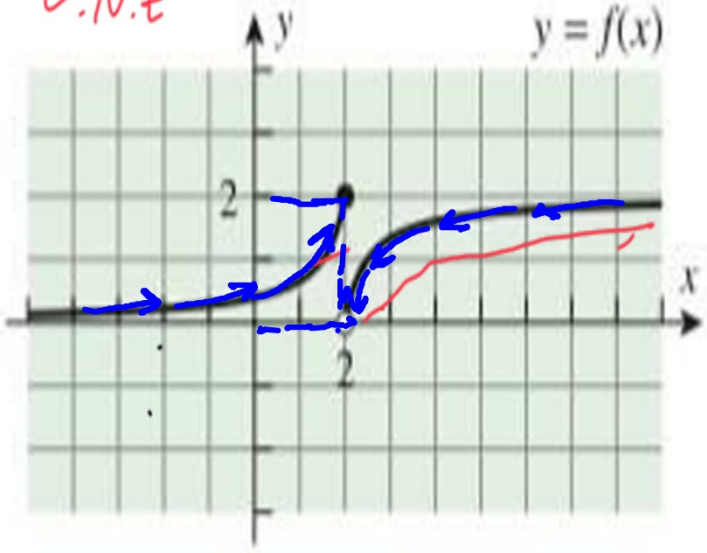


Figure Ex-4

**5.** For the function $F$ graphed in the accompanying figure, find

(a) $\lim_{x \to -2^-} F(x) = 0$ (b) $\lim_{x \to -2^+} F(x) = 0$

(c) $\lim_{x \to -2} F(x) = 0$ (d) $F(-2) = 3$

$= 0$

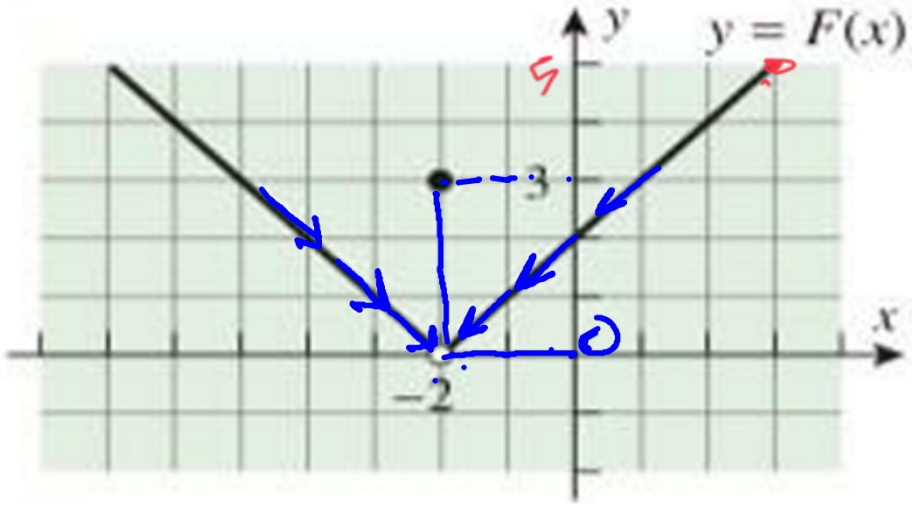


Figure E

## Properties of Limits

1. *If* $f(x) = c$ *is a constant function, then*

$$\lim_{x \to a} f(x) = \lim_{x \to a} c = c$$

ex $\lim_{x \to 20} 8 = 8$

2. $\lim_{x \to a} x^n = a^n$, *for any positive integer* $n$.

## Example

1. $\lim_{x \to 2} 7 = 7$

2. $\lim_{x \to 6} x^2 = (6)^2 = 36$

3. $\lim_{t \to -2} t^4 = (-2)^4 = 16$

## Properties of Limits

*If* $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ *exist , then:*

3. $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

4. $\lim_{x \to a}[f(x).g(x)] = \lim_{x \to a} f(x) . \lim_{x \to a} g(x)$

5. $\lim_{x \to a}[cf(x)] = c. \lim_{x \to a} f(x)$

**If $f$ is a polynomial function, then $\lim_{x \to a} f(x) = f(a)$**

## Example

Find the limits

1. $\lim_{x \to 2}(x^2 + x) = (2)^2 + 2 = 4 + 2 = 6$

2. $\lim_{q \to -1}(q^3 - q + 1) = (-1)^3 - (-1) + 1 = -1 + 1 + 1 = +1$

## Example

$$\lim_{x\to -3}(x^3+4x^2-7) = (-3)^3+4(-3)^2-7$$
$$-27+36-7=+2$$

1. $\lim_{x\to 1}\frac{2x^2+x-3}{x^3+4} = \frac{2(1)^2+1-3}{(1)^3+4} = \frac{3-3}{5} = \frac{0}{5} = 0$

2. $\lim_{t\to 4}\sqrt{t^2+1} = \sqrt{(4)^2+1} = \sqrt{16+1} = \sqrt{17}$

3. $\lim_{x\to 3}\sqrt[3]{x^2+7} = \sqrt[3]{(3)^2+7} = \sqrt[3]{16}$

4. $\lim_{x\to -1}\frac{x^2-1}{x+1} = \frac{(-1)^2-1}{-1+1} = \frac{0}{0}$ DNE

Not

$a^2-b^2=(a-b)(a+b)$

$$\lim_{x\to -1}\frac{x^2-1}{x+1} = \frac{0}{0}$$

$$\lim_{x\to -1}\frac{(x+1)(x-1)}{x+1} = \lim_{x\to -1}\frac{x-1}{1} = \frac{-1-1}{1} = -2$$

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$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \frac{0}{0}$$

$$\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2}\frac{x+2}{1} = \frac{2+2}{1} = 4$$

ex $\lim_{x\to 3} \frac{x-3}{x^2-9} = \frac{0}{0}$ $x^2-9 = x^2-3^2$

$= (x-3)(x+3)$

$$\lim_{x\to 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x\to 3} \frac{1}{x+3} = \frac{1}{6}$$